Algebra III and Trigonometry
Summer Assignment

Welcome to Algebra III and Trigonometry!

This summer assignment is a review of the skills you learned in Algebra II.

Please bring this assignment with you on the first full day of classes; it will count as a homework grade for the first marking period. Late assignments will have points deducted from the grade. After reviewing the assignment in class, expect a test based on the material.

To receive full credit for this assignment, all 50 problems (6 problems are challenge problems) must be completed with all work neatly shown on loose-leaf paper.

If you need to refresh your memory on some of the skills in this packet, examples are provided in each section.

See you in September!

EQUATIONS AND INEQUALITIES

For each problem, solve for the variable. Graph the solution sets of all inequalities.

Example A: Solve $4(2x-3)+3=4x-29$.

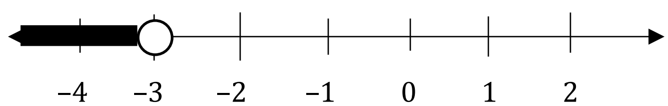
$4(2x-3)+3=4x-29$
$8x-12+3=4x-29$ Distributive Property
$8x-9=4x-29$ Simplified the left side.
$8x=4x-20$ Added 9 to each side.
$4x=-20$ Subtracted 4x from each side.
$x=-5$ Divided each side by 4.

Answer: The solution set is {–5}.

Example B: Solve $-2(y-3)+2>14$.

$-2(y-3)+2>14$
$-2y+6+2>14$ Distributive Property
$-2y+8>14$ Simplified the left side.
$-2y>6$ Subtracted 8 from each side.
$y<-3$ Divided each side by –2 and flipped the inequality sign.

Answer: The solution set is



Exercises:

1. $3x-1=5$
2. $\frac{2}{3}x=14$
3. $5x+7=2x+13$
4. $8x-3=5(2x+1)$
5. $-9+6b-b=2(6+b)$
6. $3x+1>16$
7. $\frac{3}{4}x\le\frac{3}{16}$
8. $9(x+2)\ge 72$
9. $2(x+10)\ge 55-3x$
10. $14-2x\le 4-x$

Challenge problems:

C1. $\frac{3}{8}-\frac{1}{4}x=\frac{1}{16}$

C2. $x<-4$ and $x+1>3$

FACTORING

For each problem, completely factor each polynomial, if possible.

Example C: Factor $6x^2 + 8x$. Factor out the greatest common factor, or GCF. The GCF of $6x^2$ and 8x is 2x. $6x^2 + 8x = 2x(3x + 4)$ **Answer: 2x (3x + 4)**	**Example D: Factor $16y^2 - 1$.** Use "Difference of Two Squares" formula: $a^2 - b^2 = (a + b)(a - b)$. How do we know to use this formula? The first term $16y^2$ is a square, and so is the second term 1. $16y^2 - 1 = (4y + 1)(4y - 1)$ **Answer: (4y + 1) (4y – 1)**
Example E: Factor $9m^2 - 12m + 4$. Use one of the "Perfect Trinomial Squares" formulas: $a^2 - 2ab + b^2 = (a - b)^2$. $9m^2 - 12m + 4 = (3m - 2)^2$ **Answer: $(3m – 2)^2$**	**Example F: Factor $9m^2 + 12m + 4$.** Use the other "Perfect Trinomial Squares" formula: $a^2 + 2ab + b^2 = (a + b)^2$. $9m^2 + 12m + 4 = (3m + 2)^2$ **Answer: $(3m + 2)^2$** *Notice the difference between this example and the last example.
Example G: Factor $2m^2 - 5m - 3$ by guessing and checking. The form of this polynomial doesn't match any of the formulas we used before. Guess two factors and check by FOILing them back out. $2m^2 - 5m - 3 =$ (? ?)(? ?) $(2m - 3)(m - 1)$ doesn't work $(2m + 3)(m - 1)$ doesn't work $(2m - 1)(m + 3)$ doesn't work $(2m + 1)(m - 3)$ FOILs to $2m^2 - 5m - 3$. **Answer: (2m + 1) (m – 3)**	**Example H: Factor $2x^2 - 8$.** You may need to combine two methods to completely factor out a polynomial. $2x^2 - 8 = 2(x^2 - 4)$ Factored out GCF. $= 2(x + 2)(x - 2)$ Diff. of two squares.

For each problem, use factoring to solve the equation. Make sure to move all terms to the left side of the equation so that one side of the equation equals zero!

Example I: Solve $\boldsymbol{x^2 = 3x}$.

$x^2 = 3x$

$x^2 - 3x = 0$ Subtracted 3x from both sides (so that left side of equation equals zero).

$x(x - 3) = 0$ Factored the left side.

$x = 0$ or $x - 3 = 0$ Set every factor equal to zero. In this case, there are two factors.

$x = 0$ or $x = -3$

Answer: x = 0 or x = 3.

Example J: Solve $\boldsymbol{m^2 + 4m = -4}$.

$m^2 + 4m = -4$

$m^2 + 4m + 4 = 0$ Added 4 to both sides.

$(m + 2)(m + 2) = 0$ Factored the left side.

$m + 2 = 0$ or $m + 2 = 0$ Set every factor equal to zero. In this case, there are two factors.

$m = -2$ Since both factors are the same, we can write the solution once.

Answer: m = –2.

Example K: Solve $\boldsymbol{2x^3 + 4x^2 = 6x}$.

$2x^3 + 4x^2 = 6x$

$2x^3 + 4x^2 - 6x = 0$ Subtracted 6x from both sides.

$2x(x^2 + 2x - 3) = 0$ Factored out GCF on left side.

$2x(x + 3)(x - 1) = 0$ Factored the remaining polynomial on the left side.

$2x = 0$ or $x + 3 = 0$ or $x - 1 = 0$ In this case, there are three different factors.

$x = 0$ or $x = -3$ or $x = 1$

Answer: x = 0, x = –3, or x = 1.

Exercises:

If possible, factor each polynomial completely.

11. $m^4 - 16$

12. $m^2 - 7m + 10$

13. $12 - 4y - 3x$

14. $xy^3 - 9xy$

15. $2x^2 - 2$

Solve each equation using factoring.

16. $4x^2 - 1 = 0$

17. $x^2 + x = 30$

18. $2x^2 + 9x + 4 = 0$

19. $4x^2 + 9 = 12x$

20. $12x^3 + 10x^2 - 8x = 0$

SIMPLIFYING RATIONAL EXPRESSIONS

For each expression, simplify if possible. Start by factoring the numerator and the denominator. Then divide out any common factors between the numerator and denominator.

Example L: Simplify $\frac{114}{102}$. $\frac{114}{102} = \frac{19 * 2 * 3}{17 * 2 * 3}$ $= \frac{19}{17}$ **Answer: $\frac{19}{17}$**	**Example M: Simplify $\frac{2x-4}{3x-6}$.** $\frac{2x-4}{3x-6} = \frac{2(x-2)}{3(x-2)}$ $= \frac{2}{3}$ **Answer: $\frac{2}{3}$**
Example N: Simplify $\frac{t^2-25}{2t^2-9t-5}$. $\frac{t^2-25}{2t^2-9t-5} = \frac{(t+5)(t-5)}{(2t+1)(t-5)}$ $= \frac{t+5}{2t+1}$ **Answer: $\frac{t+5}{2t+1}$**	**Example O: Simplify $\frac{3-y}{y-3}$.** $\frac{3-y}{y-3} = \frac{-1(-3+y)}{y-3}$ $= \frac{-1(y-3)}{y-3}$ $= -1$ **Answer: -1** *We don't usually think of factoring 3 – y, but we can factor out a –1 from the numerator to get a factor of y – 3 on top to match the y – 3 on the bottom.

Exercises:

If possible, simplify each expression completely.

21. $\frac{24}{72}$

22. $\frac{33}{303}$

23. $\frac{12n^9}{2n^3}$

24. $\frac{m+5}{5}$

25. $\frac{m+5}{2m+10}$

26. $\frac{4x}{x^2-x}$

27. $\frac{a^2-4}{2a^2-3a-2}$

28. $\frac{y-3}{y^2+2-12}$

29. $\frac{9s^2-25}{9s^2+30s+25}$

30. $\frac{s-t}{t-s}$

Challenge problems:

Solve each proportion using what you know.

C3. $\frac{m}{4}=\frac{4}{m}$

C4. $\frac{5}{3}=\frac{v+1}{v}$

C5. $\frac{4}{y-3}=\frac{7}{y-2}$

C6. $\frac{x}{x+1}=\frac{2x}{7}$

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

For each problem, simplify the expression by multiplying or dividing.

MULTIPLICATION	DIVISION
First, make sure there are no zeroes in any of the denominators. Multiply the numerators together to get the numerator of the product. Then multiply the denominators together to get the denominator of the product. Lastly, simplify your solution. **Example P: Simplify** $\frac{y}{4} * \frac{8}{y-1}$. $\frac{y}{4} * \frac{8}{y-1} = \frac{8y}{4(y-1)}$ $= \frac{4(2y)}{4(y-1)}$ $= \frac{2y}{y-1}$ **Answer:** $\frac{2y}{y-1}$	*First, make sure there are no zeroes in the denominators.* Rewrite division problem as a multiplication problem by changing the division sign to a multiplication sign and "flipping" the second expression. Then, follow multiplication steps. **Example Q: Simplify** $\frac{y^2}{2y^2-3y} \div \frac{y}{4y^2-9}$. $\frac{y^2}{2y^2-3y} \div \frac{y}{4y^2-9} = \frac{y^2}{2y^2-3y} * \frac{4y^2-9}{y}$ $= \frac{y^2(4y^2-9)}{y(2y^2-3y)}$ $= \frac{y^2(2y+3)(2y-3)}{y * y(2y-3)}$ $= 2y+3$ **Answer:** $2y+3$

Exercises:

Express each product or quotient in simplest form.

31. $\frac{1}{18y} * 2y$

32. $\frac{m^2n}{n^2p} * \frac{p}{q}$

33. $\frac{x+1}{3} * \frac{4}{x}$

34. $\frac{2n-4}{5} * \frac{10}{n-2}$

35. $\frac{a+b}{14} * \frac{7}{b+a}$

36. $\frac{x^2}{y^2} \div \frac{x^3}{a}$

37. $\frac{r^3}{2s} \div \frac{-r^4}{8s}$

38. $\frac{1-r}{3} \div \frac{r-1}{2}$

39. $\frac{4-x^2}{10} \div \frac{x-4}{5}$

40. $\frac{t^2-36}{4t} \div (6-t)$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

For each problem, simplify the expression by adding and/or subtracting.

Find a common denominator.

Example R: Simplify $\frac{4x+1}{3}-\frac{x-5}{2}$.

$$\frac{4x+1}{3}-\frac{x-5}{2}=\frac{2(4x+1)}{2\cdot 3}-\frac{3(x-5)}{3\cdot 2}$$

$$=\frac{8x+2}{6}-\frac{3x-15}{6}$$

$$=\frac{8x+2-(3x-15)}{6}$$

$$=\frac{8x+2-3x+15}{6}$$

$$=\frac{5x+17}{6}$$

Answer: $\frac{5x+17}{6}$

It may be necessary to factor the denominators first. Then find a common denominator.

Example S: Simplify $\frac{x+1}{x^2-2x}-\frac{6}{x^2-4}$.

$$\frac{x+1}{x^2-2x}-\frac{6}{x^2-4}=\frac{x+1}{x(x-2)}-\frac{6}{(\mathrm{x}-2)(x+2)}$$

$$=\frac{(x+1)(x+2)}{x(x-2)(x+2)}-\frac{6(x)}{x(x-2)(x+2)}$$

$$=\frac{x^2+3x+2-6x}{x(x-2)(x+2)}$$

$$=\frac{x^2-3x+2}{x(x-2)(x+2)}$$

$$=\frac{(x-2)(x-1)}{x(x-2)(x+2)}$$

$$=\frac{x-1}{x(x+2)}$$

Answer: $\frac{x-1}{x(x+2)}$ **or** $\frac{x-1}{x^2-2x}$

Exercises:

Express each sum or difference in simplest form.

41. $\frac{2x}{3} + \frac{x+1}{6}$

42. $\frac{4x}{7y} + \frac{3y}{21y}$

43. $\frac{y}{y-4} - \frac{32}{y^2-16}$

44. $\frac{b+2}{b-3} - \frac{15}{b^2-3b}$

45. $\frac{2x}{2x+6} + \frac{5x}{x+3}$

46. $\frac{4}{x+y} - \frac{7}{x-y}$

47. $y - 1 + \frac{1}{y+1}$

48. $\frac{b-2}{b-3} - \frac{b-4}{b-5}$

49. $\frac{t}{t+3} + \frac{18}{9-t^2}$

50. $\frac{-4y}{y^2-4} - \frac{y}{y+2}$